Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

15 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Let $f(x) = \ln x - 5x$, for $x > 0$.

(a) Find $f'(x)$. [2]

(b) Find $f''(x)$. [1]

(c) Solve $f'(x) = f''(x)$. [2]

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2. [Maximum mark: 6]

A biased four-sided die is rolled. The following table gives the probability of each score.

Score	1	2	3	4
Probability	0.28	k	0.15	0.3

(a) Find the value of k. [2]

(b) Calculate the expected value of the score. [2]

(c) The die is rolled 80 times. On how many rolls would you expect to obtain a three? [2]

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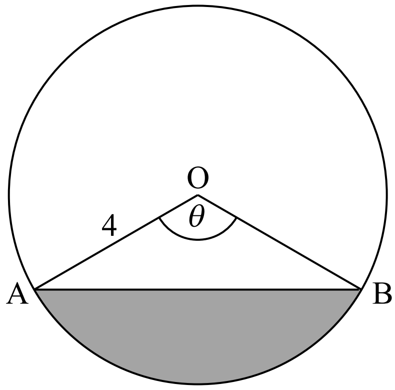
3. [Maximum mark: 6]

The diagram shows a circle, centre O, with radius $4\,\text{cm}$. Points A and B lie on the circumference of the circle and $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 \le \theta \le \pi$.

diagram not to scale



(a) Find the area of the shaded region, in terms of θ. [3]

(b) The area of the shaded region is $12\,\text{cm}^2$. Find the value of θ. [3]

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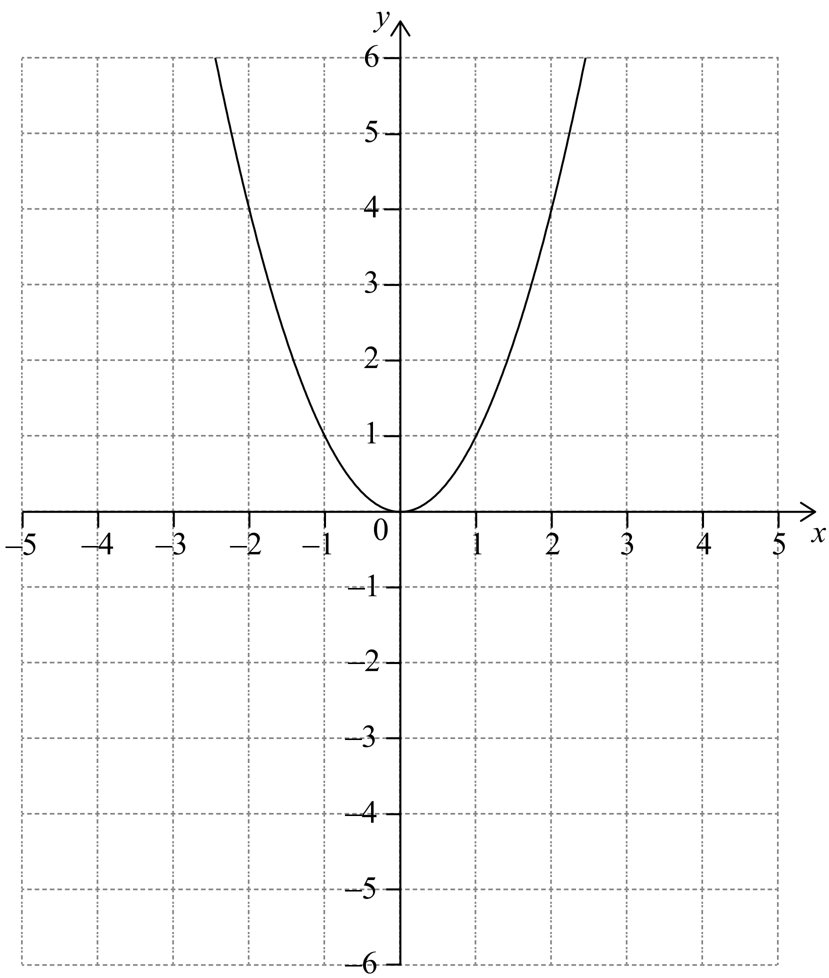
4. [Maximum mark: 7]

Let $g(x) = -(x-1)^2 + 5$.

(a) Write down the coordinates of the vertex of the graph of g. [1]

Let $f(x) = x^2$. The following diagram shows part of the graph of f.



The graph of g intersects the graph of f at $x = -1$ and $x = 2$.

(b) On the grid above, sketch the graph of g for $-2 \le x \le 4$. [3]

(c) Find the area of the region enclosed by the graphs of f and g. [3]

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(Question 4 continued)

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5. [Maximum mark: 6]

Two events A and B are such that $\mathrm{P}(A) = 0.62$ and $\mathrm{P}(A \cap B) = 0.18$.

(a) Find $\mathrm{P}(A \cap B')$. [2]

(b) Given that $\mathrm{P}\left((A \cup B)'\right) = 0.19$, find $\mathrm{P}(A \mid B')$. [4]

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6. [Maximum mark: 7]

Triangle ABC has $a = 8.1\,\text{cm}$, $b = 12.3\,\text{cm}$ and area $15\,\text{cm}^2$. Find the largest possible perimeter of triangle ABC.

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7. [Maximum mark: 8]

Let $f(x) = \mathrm{e}^{2\sin\left(\frac{\pi x}{2}\right)}$, for $x > 0$.

The kth maximum point on the graph of f has x-coordinate x_k where $k \in \mathbb{Z}^+$.

(a) Given that $x_{k+1} = x_k + a$, find a. [4]

(b) Hence find the value of n such that $\sum_{k=1}^{n} x_k = 861$. [4]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

The following table shows values of $\ln x$ and $\ln y$.

ln *x*	1.10	2.08	4.30	6.03
ln *y*	5.63	5.22	4.18	3.41

The relationship between $\ln x$ and $\ln y$ can be modelled by the regression equation $\ln y = a \ln x + b$.

(a) Find the value of a and of b. [3]

(b) Use the regression equation to estimate the value of y when $x = 3.57$. [3]

The relationship between x and y can be modelled using the formula $y = kx^n$, where $k \neq 0$, $n \neq 0$, $n \neq 1$.

(c) By expressing $\ln y$ in terms of $\ln x$, find the value of n and of k. [7]

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9. [Maximum mark: 17]

The weights, in grams, of oranges grown in an orchard, are normally distributed with a mean of $297\,\text{g}$. It is known that 79% of the oranges weigh more than $289\,\text{g}$ and 9.5% of the oranges weigh more than $310\,\text{g}$.

(a) Find the probability that an orange weighs between $289\,\text{g}$ and $310\,\text{g}$. [2]

The weights of the oranges have a standard deviation of σ.

(b) (i) Find the standardized value for $289\,\text{g}$.

(ii) Hence, find the value of σ. [5]

The grocer at a local grocery store will buy the oranges whose weights exceed the 35th percentile.

(c) To the nearest gram, find the minimum weight of an orange that the grocer will buy. [3]

The orchard packs oranges in boxes of 36.

(d) Find the probability that the grocer buys more than half the oranges in a box selected at random. [5]

The grocer selects two boxes at random.

(e) Find the probability that the grocer buys more than half the oranges in each box. [2]

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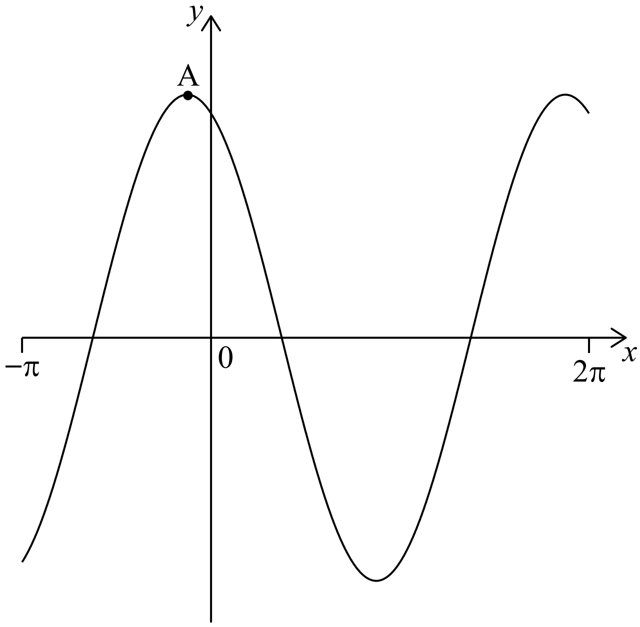
10. [Maximum mark: 15]

Let $f(x) = 12\cos x - 5\sin x$, $-\pi \le x \le 2\pi$, be a periodic function with $f(x) = f(x + 2\pi)$.
The following diagram shows the graph of f.



There is a maximum point at A. The minimum value of f is -13.

(a) Find the coordinates of A. [2]

(b) For the graph of f, write down

(i) the amplitude;

(ii) the period. [2]

(c) Hence, write $f(x)$ in the form $p\cos(x + r)$. [3]

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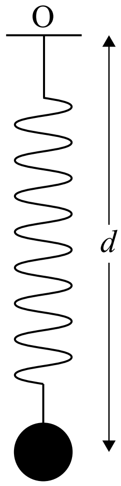
(Question 10 continued)

A ball on a spring is attached to a fixed point O. The ball is then pulled down and released, so that it moves back and forth vertically.

diagram not to scale



The distance, d centimetres, of the centre of the ball from O at time t seconds, is given by

$$d(t) = f(t) + 17,\ 0 \le t \le 5.$$

(d) Find the maximum speed of the ball. [3]

(e) Find the first time when the ball's speed is changing at a rate of $2\,\text{cm}\,\text{s}^{-2}$. [5]

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